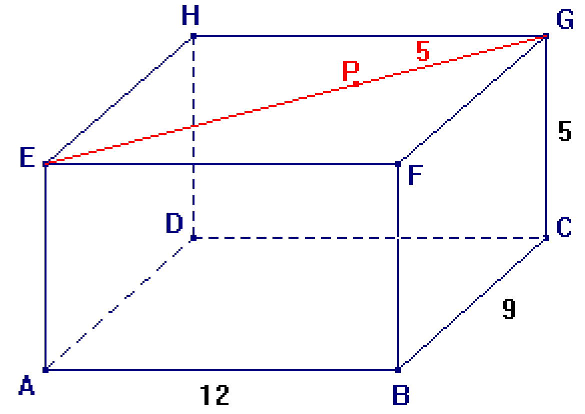
Afstanden en hoeken in de ruimte
Oplossing oefening 4



Geg.: Balk $\begin{pmatrix} EFGH \\ ABCD \end{pmatrix}$

$|AB| = 12$; $|BC| = 9$; $|CG| = 5$

$P \in [EG]$, $|PG| = 5$

Gevr.: $B\hat{P}E$

Opl.:

- Berekening |EG| met stelling van Pythagoras in bovenvlak van de balk

$|EG|^2 = |EF|^2 + |FG|^2 \quad \Rightarrow |EG|^2 = 12^2 + 9^2 = 144 + 81 = 225$

$\Rightarrow |EG| = \sqrt{225} = 15$

Hieruit volgt $|EP| = |EG| - |PG| = 15 - 5 = 10$

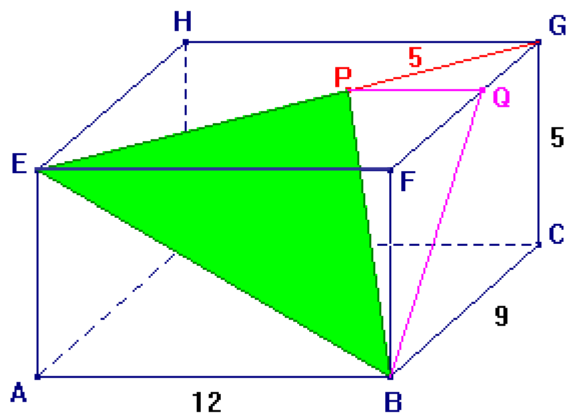
- Berekening |EB| met stelling van Pythagoras in voorvlak van de balk

$|EB|^2 = |EA|^2 + |AB|^2 \quad \Rightarrow |EB|^2 = 5^2 + 12^2 = 25 + 144 = 169$

$\Rightarrow |EB| = \sqrt{169} = 13$

- Als we nu |BP| nog kunnen bepalen kunnen we de gevraagde hoek berekenen met de cosinusregel.

- Hiertoe construeren we $Q \in [FG]$ zodat PQ // EF



$\Delta EFG \sim \Delta PQG \quad \Rightarrow \frac{|PQ|}{|EF|} = \frac{|PG|}{|EG|} = \frac{|QG|}{|FG|}$

$\Rightarrow \frac{|PQ|}{12} = \frac{5}{15} = \frac{|QG|}{9}$

$\Rightarrow |PQ| = 4$ en $|QG| = 3$

$|FQ| = |FG| - |QG| = 9 - 3 = 6$

$|BQ|^2 = |BF|^2 + |FQ|^2 \quad \Rightarrow |BQ|^2 = 5^2 + 6^2 = 25 + 36 = 61$

$\Rightarrow |BQ| = \sqrt{61}$

ΔBPQ is rechthoekig in Q, dus kunnen we BP berekenen met Pythagoras:

$|BP|^2 = |BQ|^2 + |PQ|^2 \quad \Rightarrow |BP|^2 = (\sqrt{61})^2 + 4^2 = 61 + 16 = 77$

$\Rightarrow |BP| = \sqrt{77}$

- Berekening van $B\hat{P}E$ met de cosinusregel in ΔBPE:

$|BE|^2 = |BP|^2 + |EP|^2 - 2.|BP|.|EP|.\cos(B\hat{P}E) \quad \Rightarrow \cos(B\hat{P}E) = \frac{|BE|^2 - |BP|^2 - |EP|^2}{-2.|BP|.|EP|}$

Dus $\cos(B\hat{P}E) = \frac{169 - 100 - 77}{-2.10.\sqrt{77}} = 0{,}04558 \quad \Rightarrow B\hat{P}E = 87° \, 23' \, 14''$